DYSON-PHILLIPS EXPANSION METHOD TO SOLVE INTEGRO-DIFFERENTIAL EQUATIONS

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ABSTRACT

In this work we consider differential-integral equations such as

$$\frac{du(t)}{dt} = au(t) + \int_0^1 k(t,s)u(s)ds\,, \tag{1}$$

or equations of the following type furnished by linear transport theory

$$\frac{\partial u(x,\mu,t)}{\partial t} + \mu\frac{\partial u(x,\mu,t)}{\partial x} + h(x,\mu)u(x,\mu,t) = \int_{-1}^{1} k(x,\mu,\mu')u(x,\mu',t)d\mu' + q(x,\mu,t)\,, \tag{2}$$

where in both equations x, μ and t represent independent variables, u the unknown function and k is the kernel. Certainly the function q is the source function. Both equations can put in the form

$$\frac{du(t)}{dt} = Au(t) + Bu(t)\,, \tag{3}$$

where A is the generator of the semigroup $W_0(t)$ and $A + B$ is the generator of the semigroup $W(t)$. The Dyson-Phillips takes the form [1,2]

$$Wt) = \sum_{n=0}^{\infty} W_0^{(n)}(t)\,, \qquad t \geq 0\,, \tag{4}$$

where

$$\begin{aligned} W_0^{(0)}(t) &= W_0(t) \\ W_0^{(n)}(t) &= -\int_0^t W_0(t-s)BW_0^{(n-1)}(s)ds \end{aligned} \tag{5}$$

We solve both problems above with some boundary conditions and give numerical results obtained with a computer algebraic system. Some extensions are obtained in the framework of the strongly continuous semigroup theory [3].

REFERENCES

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